Exploring the Impact of Thermal Effects on Vibration and Buckling in Quasi-Ply Laminates with Cutouts: A Comprehensive Analysis

Shivakumara N V[1], K S Subash Chandra[2], Mamatha A[3], Maharudra[4]

[1,2,3,4] Government Polytechnic, Department of Technical Education, Bengaluru 560 001, Karnataka, India

Abstract. This research delves into the intricate relationship between temperature variations and the vibration and buckling characteristics of quasi-isotropic composite laminates featuring a centrally positioned circular cutout. Employing a sophisticated quadratic isoparametric nine-noded heterosis element within the finite element formulation, we meticulously crafted a MATLAB program to meticulously examine a range of parameters. These include the effects of thickness variations, diverse boundary conditions, and variations in cutout sizes across fluctuating thermal environments. Our comprehensive analysis unveils a compelling narrative: the vulnerability of thin laminated panels with cutouts to temperature differentials emerges as a pivotal finding. These temperature gradients exert a pronounced influence, manifesting in substantial alterations to both vibration and buckling behavior. By illuminating these intricate dynamics, our study sheds invaluable light on the nuanced interplay between material composition, geometric features, and environmental factors in composite laminate performance.

Keywords: Temperature, Vibration, Buckling, Composite, Cutouts, Heterosis

1 Introduction

Laminated composites, composed of layers of orthotropic materials, offer numerous advantages such as reduced self-weight, high specific strength, excellent fatigue resistance, high stiffness-to-weight ratios, and versatile design options. These properties have led to their widespread adoption in various fields including maritime, civil engineering, aerospace, military, etc. However, these structures are often subjected to environmental factors, such as temperature variations, which can have significant effects. Heat conduction within laminated composites can lead to expansion or contraction, particularly affecting matrix-dominated composites. An increase in temperature tends to cause swelling or expansion in the matrix, while a decrease results in contraction. Although the fibers within the matrix are less sensitive to thermal variations compared to the matrix material, dimensional changes in laminates can still occur. These dimensional transformations can lead to the development of thermal stresses or residual stresses, the latter being introduced during the fabrication process. In addition to temperature variations, changes in specific heat due to variations in density and thermal expansion coefficients can further contribute to the structural integrity issues. As a result, variations in temperature often contribute to the failure of engineering structures. Laminated composite structures are particularly susceptible to failure due to buckling, severe vibration amplitudes, or excessive stress levels under thermal or thermo-mechanical conditions.

Studies in the literature have focused on analyzing the vibration and buckling behavior of laminated composite structures under hygrothermal environments. These investigations aim to understand the complex interactions between material properties, environmental conditions, and structural responses to ensure the reliability and safety of laminated composite structures in real-world applications. Whitney and Ashton [2] explored the influence of environmental factors on the free vibration of laminated plates using the Ritz technique, examining both symmetric and asymmetric laminates based on classical laminated plate theory.

Ram and Sinha [3,4] incorporated rotary inertia and transverse shear deformation effects following the Yang-Norris-Stavsky approach within the Mindlin plate theory to analyze symmetric and anti-symmetric laminated composite plates under hygrothermal conditions. They utilized an eight-node isoparametric finite element with five degrees of freedom per node for their analysis. Naidu and Sinha [5] integrated transverse shear deformation effects, including geometric non-linearity, within the First-order Shear Deformation Theory (FSDT) to predict the free vibration thermal response of cylindrical, spherical, and hyperboloid laminated shells. They employed an eight-node isoparametric quadrilateral element with six degrees of freedom per node. Biswal et al. [6,7] conducted experimental and analytical investigations on composite shells subjected to thermal conditions, utilizing FSDT with an eight-noded doubly curved isoparametric element featuring five degrees of freedom per node.

Patel et al. [8] employed the Higher Order Shear Deformation Theory (HSDT) to predict the linear free vibration response of thick laminated plates under the influence of moisture and temperature. They used a C0 eight-noded quadrilateral serendipity plate element with thirteen degrees of freedom per node for their analysis. HSDT was implemented using an eight-noded quadratic finite element with nine degrees of freedom per node. Padhi and Pandit [9] introduced the Higher Order Zigzag Theory (HOZT) for the analysis of laminated composite and sandwich plates under static and free vibration conditions, accounting for the effects of moisture and temperature changes. They utilized a nine-noded isoparametric finite element model with eleven degrees of freedom per node for their analysis, incorporating stress-free conditions at the top and bottom plates and continuity conditions at interfaces.

Cutouts, commonly used for access to pipelines, fuselage, doors, and windows, are highly susceptible to buckling even at extremely low stress levels. Thermo-mechanical analysis of such cutouts is essential as they have free edges exposed to the environment without edge constraints.

Most of the studies are done for plates and shells considering without any cutouts. Only a few studies are done to carry out thermal effect on laminate with cutouts. In the present study the effect of temperature on circular cutouts is analyzed for vibration and buckling of laminated composite panels for the quasi-isotropic symmetric laminate of 8 layers. A quadratic isoparametric nine-noded heterosis element is used in the present analysis.

2 Theoretical Formulation

Utilizing the postulates of the Reissner-Mindlin hypothesis, the current model under investigation integthe shear deformation effect. The displacement field is depicted as follows: rates

$$\{\overline{u}, \overline{v}, \overline{w}\}^T = \{u(x,y), v(x,y), w(x,y)\}^T + \{z\theta_x, z\theta_y, 0\}^T \qquad (1)$$

The formulation employs Green-Lagrange's strain-displacement relationship extensively. This relationship comprises two components: (i) linear strain terms, which are applied in deriving the elastic stiffness matrix, and (ii) nonlinear strain terms, utilized for the geometrical stiffness matrix.

$$\{\varepsilon\} = \{\varepsilon_l\} + \{\varepsilon_{nl}\} \tag{2}$$

The linear strains associated with the displacement given in Eq. (2) can be formally expressed as:

$$\left.\begin{aligned} \varepsilon_{xl} = \frac{\partial u}{\partial x} + zk_x, \quad \varepsilon_{yl} = \frac{\partial v}{\partial y} + zk_y, \quad \gamma_{xyl} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + zk_{xy}, \\ \gamma_{xzl} = \frac{\partial w}{\partial x} + \theta_x, \quad \gamma_{yzl} = \frac{\partial w}{\partial y} + \theta_y \end{aligned}\right\} \tag{3}$$

where bending strains are

$$k_x = \frac{\partial \theta_x}{\partial x}, \quad k_y = \frac{\partial \theta_y}{\partial y}, \quad k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \tag{4}$$

The non-linear strain components according to Green's strains are

$$\left.\begin{aligned} \varepsilon_{xnl} &= \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right] \\ \varepsilon_{ynl} &= \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right] \\ \gamma_{xynl} &= \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w_p}{\partial y}\right) + \\ & z^2\left[\frac{\partial \theta_x}{\partial x}\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\frac{\partial \theta_y}{\partial y}\right] \end{aligned}\right\} \tag{5}$$

2.1 Constitutive Relations

The constitutive relations governing the behavior of the composite panel under temperature and moisture conditions, as applied in the current buckling analysis, align with findings from prior research [7].

$$\{F\} = [D]\{\varepsilon\} - \{F^N\} \tag{6}$$

where, $\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$,

$\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}, \kappa_x, \kappa_y, \kappa_{xy}, \phi_x, \phi_y\}^T$ and

$\{F^N\} = \{N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, 0, 0\}^T$

where $\{F^N\}$ is the non- mechanical force due to thermal loads and $N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N$ are non-mechanical force and moment resultants per unit length due to temperature.

2.2 Finite Element Analysis

In the present analysis under hygrothermal loading, a nine-noded isoparametric heterosis element is utilized, incorporating five degrees of freedom (u, v, w, θx, and θy) at each node. This element combines features from both an 8-noded serendipity and a 9-noded Lagrange element. Specifically, it comprises five degrees of freedom at all edge nodes (u, v, w, θx, and θy), while the inner node possesses four degrees of freedom, as depicted in Fig. 1.

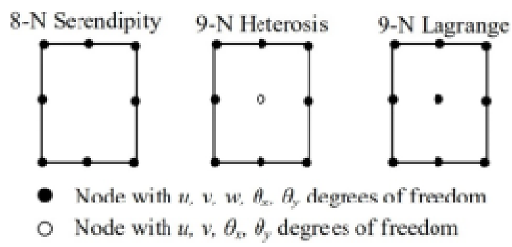

Figure 1. Plate elements of different kinds.

The distinct matrices involved in analyzing the plate element are as follows:

$$[M_e] = \iint_{-1}^{+1} [N]^T[\rho][N]|J|\, d\xi d\eta \tag{7}$$

$$[K_e] = \iint_{-1}^{+1} [B]^T[D][B]|J|\, d\xi d\eta \tag{8}$$

$$[K_{\sigma e}^{a}] = \iint_{-1}^{+1} [H]^T[P][H]|J|\, d\xi d\eta \tag{9}$$

The equations (7-9) represent the computation of the element mass matrix, the element elastic stiffness matrix, and the element geometric stiffness matrix due to applied in-plane loads of the element. These matrices are calculated using the principle of minimum potential energy.

2.3 Governing equation

The process involves assembling the element matrices to obtain their corresponding global matrices: [K] representing the stiffness matrix, [K_initial] for the initial stress stiffness matrix under hygrothermal conditions, and [M] for the mass matrix excluding damping effects. The solution methodology encompasses steps aimed at determining the natural frequencies by fulfilling the given condition.

$$\left|\left[[K] + [K_\sigma^r]\right] - \omega^2[M]\right| = 0 \tag{12}$$

Linear buckling loads are deduced from fulfilling the mentioned condition

$$\left|\left[[K] + [K_\sigma^r]\right] - P_{cr}[K_\sigma^a]\right| = 0 \tag{13}$$

Equations (12) to (13) present the generalized eigenvalue problem, where 'ω' represents the natural frequency, and the critical load is denoted by 'Pcr.' These equations are solved using MATLAB.

3 Problem definition

The fundamental configurations of the problems under consideration are depicted in Fig. 2(a), which includes displacement coordinates relative to the x, y, and z-axes, respectively.

Aimed at the cutout laminated panels, the mesh pattern followed is shown in Fig. 2(b).

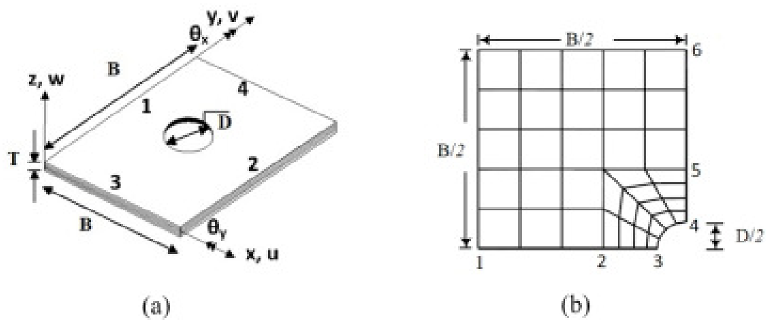

Figure 2. (a) Geometry of the Laminated panel (b) Mesh pattern of cutout panel.

The boundary conditions are represented by the letters "S" (simply supported) and "C" (clamped), arranged in a specific sequence to denote various boundary conditions. For simplicity, the abbreviation "SSCC" is utilized, where the first two consecutive letters indicate a simply supported condition at x = 0 and x = a, while the remaining two letters indicate a clamped condition at y = 0 and y = b, respectively. This sequence follows the order of edge numbers as illustrated in Fig. 2(a). In this study, the assessment of buckling effects occurs in two distinct stages. Initially, a pre-buckling analysis is performed to determine the in-plane stress distributions of the plate element. Subsequently, the critical buckling loads are calculated using the pre-buckling stresses.

The displacement boundary conditions considered for both pre-buckling and buckling analyses are detailed as follows:

(1) Simply supported condition (SSSS):

(i) For pre-buckling stress analysis:

- At x = 0 and x = a: w = θy = 0
- At y = 0 and y = b: w = θx = 0
- At nodes along the edges x = a/2 and y = b/2: u = 0, v = 0

(ii) For buckling analysis:

- Along x = 0 and x = a: u = w = θy = 0
- Along y = 0 and y = b: v = w = θx = 0

(2) Clamped condition (CCCC):

(i) For pre-buckling stress analysis:

- Along x = 0 and x = a: w = θx = θy = 0
- Along y = 0 and y = b: w = θx = θy = 0
- At nodes along the edges x = a/2 and y = b/2: u = 0, v = 0

(ii) For buckling analysis:

- Along x = 0 and x = a: u = v = w = θx = θy = 0
- Along y = 0 and y = b: u = v = w = θx = θy = 0

The vibration frequency and critical loads are expressed in non-dimensional form as follows:

Non-dimensional frequency, $$\bar{\omega} = \omega_{abs} B^2 \left(\frac{\rho}{E_{22} T^2}\right) \quad (3)$$

Non-dimensional critical load, $$N_{xcr} = \frac{P_{cr} B^2}{E_{22} T^3} \quad (4)$$

4 RESULTS AND DISCUSSION

In the current investigation, quasi-isotropic (+45/-45/0/90)S eight-layered laminates are analyzed under uniform temperature distribution for buckling analysis and without external static load for vibrational results. Various configurations of graphite/epoxy laminates with differing side-to-thickness ratios, boundary conditions, and cutout sizes are examined. Material properties at elevated temperatures, utilized in this analysis, are outlined in Table 1. These properties are assumed to remain constant throughout the study, implying they do not vary with temperature.

Table 1. The elastic moduli of graphite/epoxy at various temperatures are as follows: G13 = G12, G23 = 0.5G12, υ12 = 0.3, α1 = -0.3 x 10^-6/K, and α2 = 28.1 x 10^-6/K.

Elastic Moduli (GPa)	Temperature T (K)					
	300	325	350	375	400	425
E_1	130	130	130	130	130	130
E_2	9.50	8.50	8.00	7.50	7.00	6.75
G_{12}	6.0	6.0	5.5	5.0	4.75	4.5

4.1 Effect of Temperature on Vibration Characteristic for different Thickness, Boundary Conditions and Cutout Size.

The variation of frequencies of the laminated panel subjected to different temperatures for different thickness ratio (B/T), cutout ratio (D/B) and boundary conditions are computed using present FE formulation. Fig. 3 indicates that the buckling behavior of thick laminates with a cutout is almost unchanged due to the presence of temperature gradient, while the thin laminates are very much affected. Fig. 4 shows the nonlinear behaviour observed in the thin cutout panel. Fig. 5 and Fig. 6 shows that there is an increase in frequencies when the temperature is increased with an increase in B / T and boundary constraints.

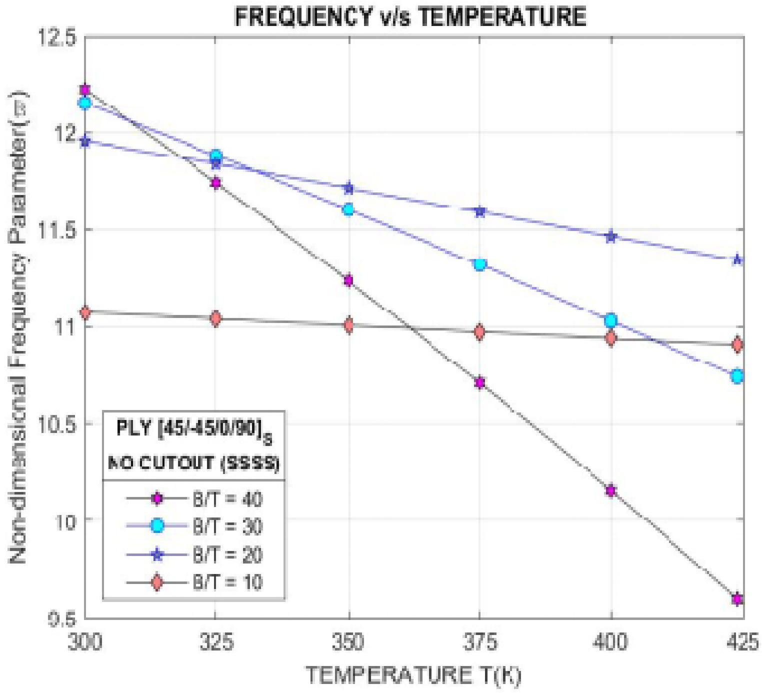

Figure 3. The effect of plate thickness on the frequency of a simply supported plate composed of (45/-45/0/90)S laminate, subjected to temperature variations without any cutout, is under investigation.

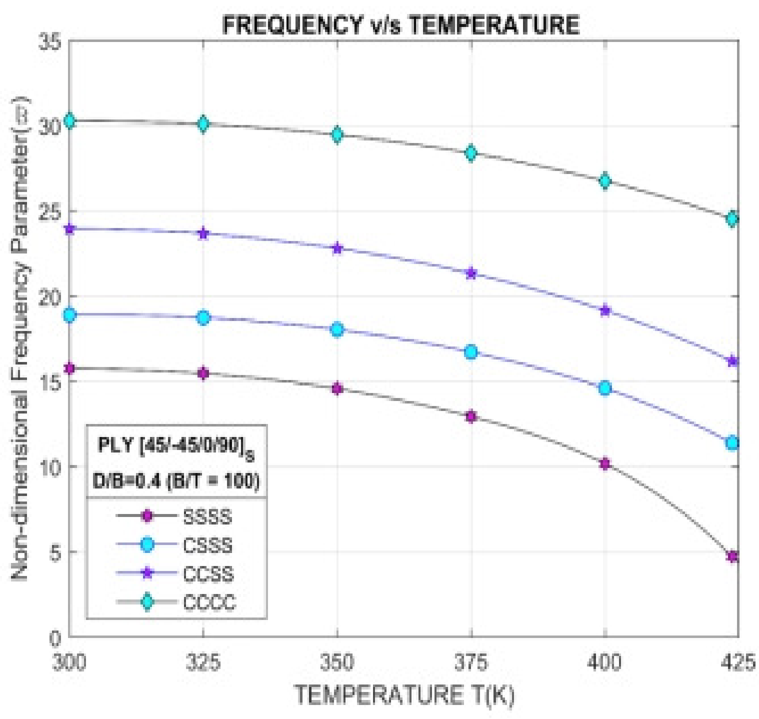

Figure 4. Variation in frequency as a function of temperature T (K) for the centrally located circular cutout panel (D/B=0.4) and thickness ratio (B/T=100) for various boundary condition

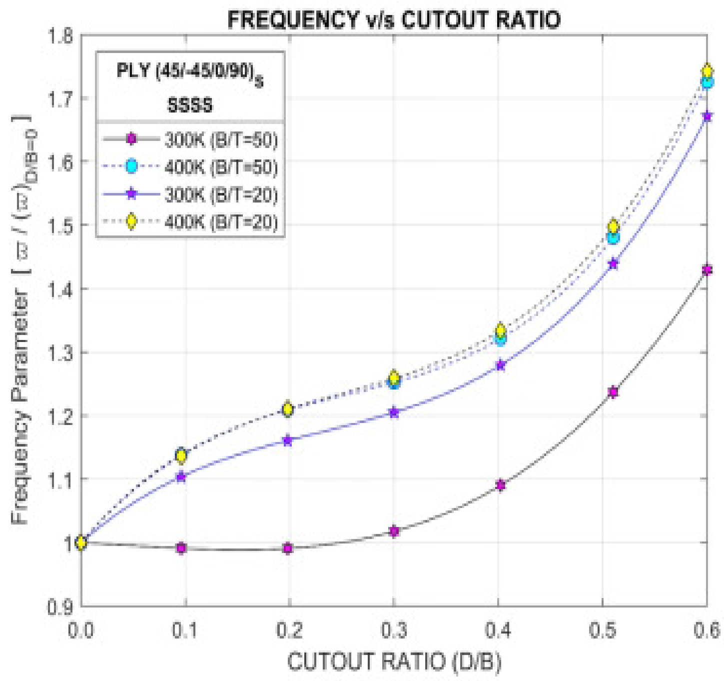

Figure 5. Influence of temperature and thickness ratio on vibration for cutout panels with simply supported boundary condition.

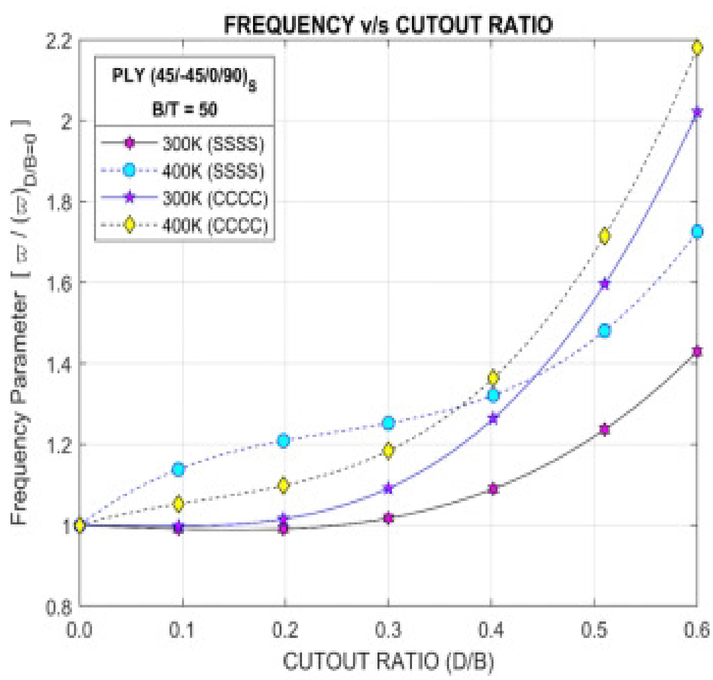

Figure 6. Influence of temperature and boundary condition on vibration for cutout panels with thickness ratio (B/T) 50.

5 Conclusion

The investigation conducted in this study delves into the influence of varying temperature degrees on the vibration and buckling characteristics of composite laminates, with and without cutouts. Summarizing the significant outcomes of this research:

- The frequency of vibration decreases with the increase in temperature for any thickness of plate. However, the decrease in frequency is significant for thin plates
- The frequency parameter increases with the increase in cutout size. On the other hand, the buckling parameter decreases with the increase in cutout size.
- Analysis indicates that thin plates may buckle merely due to small variation on temperature.

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